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THE PLANE PROBLEM FOR A CRACK BETWEEN TWO LINEARLY ELASTIC MEDIA†

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An exact solution of the plane problem of the non-linear theory of elasticity is constructed for a crack at the interface between two different media for an elastic potential corresponding to a linearly elastic prestressed material. A unified version [1] of the plane problem of the non-linear theory of elasticity is used. For comparison the solution of the corresponding problem for the linear theory with the same elastic potential is given. The asymptotic forms of the stresses in both cases are compared. It is shown that the asymptotic form of the components of the symmetric Biot tensor is of the same order as the asymptotic form of the stresses in the linear problem, but in this case the first has no oscillations while the squares of the coefficients of the principal terms of the asymptotic forms of these stresses are identical, apart from a factor, with the Rice–Cherepanov integral. The displacements in the non-linear problem do not oscillate only when a certain relation for the elasticity constants of the contacting media is satisfied.

The solution of the plane problem of the linear theory of elasticity for an interphase crack [2–6] contains an oscillating singularity and therefore loses meaning on certain parts of the surface of the crack near its tips. The correction of this solution by introducing additional contact conditions on the crack surface [7–12] removes the oscillation but preserves the singularity of the stresses and strains. Hence, in both cases the linear theory of elasticity is untenable in a certain neighbourhood of the crack tip, and this requires that the corresponding problems must be solved in the non-linear formulation.

1. FORMULATION OF THE PROBLEM

Consider the simultaneous deformation of two different media under plane strain conditions for a plane stressed state. In the x_1^0, x_2^0 plane boundary L between the two media in the undeformed state coincides with the real axis $x_2^0=0$, on which the crack is situated $L_1=\{x_1^0, x_2^0: |x_1^0|<1,\ x_2^0=0\}$. We will assume that the crack surfaces are free from external forces, and the principal stresses $\sigma_j^{0k\infty}=Q_j^k$ $(j=1, 2)$, act at infinity in each medium S_k $(k=1, 2)$ and Q_1^k makes an angle of χ_k with the x_1^0 axis (Fig. 1).

We will take the law of elasticity for the material of each medium in the form

$$\sigma_j^0=\sigma_k^\upsilon\xi^+-(-1)^j\alpha_k^\upsilon\xi^-+\sigma_k^r \qquad (j,k=1,2) \tag{1.1}$$

$$\xi^\pm=\tfrac{1}{2}[(\lambda_1-1)\pm(\lambda_2-1)]$$

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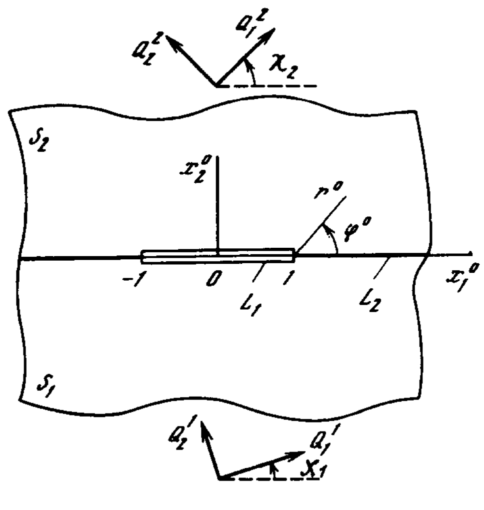

Fig. 1.

where σ_j^0 are the principal values of the Biot tensor (the principal stresses), the $\lambda_j - 1$ are the principal relative extensions, σ_j^υ, α_k^υ are the elasticity constants of the medium S_k, the values of which, generally speaking, vary when the constant σ_k' (the residual stress in the reference configuration of the corresponding medium) varies. Quantities with the superscript 0 relate to the undeformed (the reference) configuration while those without this superscript relate to the deformed (actual) configuration.

For small deformations and angles of rotation, relations (1.1) reduce to Hooke's law along the principal axes $\sigma_j^0 = \sigma_j$, $\xi^\pm = \frac{1}{2}(e_1 \pm e_2))$ and $\sigma_k^\upsilon = 2(\lambda_k^\upsilon + \mu_k^\upsilon)$, $\alpha_k^\upsilon = 2\mu_k^\upsilon$, where when $\sigma_k' = 0$, $\lambda_k^\upsilon = \lambda_{(k)}$, $\mu_k^\upsilon = \mu_{(k)}$ are Lamé constants, and for a plane stressed state λ_k^υ must be replaced by $\lambda_k^{\upsilon *} = 2\lambda_k^\upsilon \mu_k^\upsilon / (\lambda_k^\upsilon + 2\mu_k^\upsilon)$.

The linear relations (1.1) between the quantities σ_j^0 and $\lambda_j - 1$ which form an energy pair [1], correspond to linearly elastic prestressed materials for which when $\lambda_1 \equiv \lambda_2 = 1$, i.e. when there is no strain, $\sigma_1^0 = \sigma_2^0 = \sigma_k'$.

We will construct a solution of the non-linear problem in the special case when, for each medium S_k, the value of the preliminary uniform stress (the residual stress) is identical with the corresponding modulus of elasticity of the material, i.e. $\sigma_k' = \sigma_k^\upsilon = \sigma_k^*$ and $\alpha_k^\upsilon = \alpha_k$. The analysis of the singularities of the stress–strain state in the neighbourhood of the crack tip is then simplified since the solution of the problem can be obtained in analytic form.

This does not mean that the solution of the problem of the non-linear theory of elasticity constructed below has a particularly special form. It can be seen from the solution of the non-linear problem obtained that the stress–strain state around the crack tip does not change qualitatively when the elastic constants change. As in the linear problem, the dependence of the asymptotic formulae of the solution of the non-linear problem on the elastic constants is a characteristic feature of an interface crack and is expressed in the relationship between the bi-elastic constant ε and σ_k^* and α_k $(k = 1, 2)$. For a crack in a uniform medium (where both media are the same) this relation naturally disappears.

We must add to the above the fact that the linearity of relations (1.1) enables us to investigate in "pure form" the effect of a geometric non-linearity in plane problems of the theory of elasticity.

2. FUNDAMENTAL RELATIONS

The elastic potential

$$W=\sigma_k^*|\partial z/\partial\zeta|^2+\alpha_k|\partial z/\partial\bar{\zeta}|^2 \qquad (\sigma_k^*\leqslant\alpha_k) \tag{2.1}$$

corresponds to relation (1.1) when $\sigma_k^r=\sigma_k^\upsilon=\sigma_k^*$, $\alpha_k^\upsilon=\alpha_k$. In this case the solution of the plane problems of the non-linear theory of elasticity reduces [1, 13] to finding the complex piecewise-holomorphic functions $\Phi(\zeta)$ and $\Psi(\zeta)$, which satisfy the linear boundary conditions either for the stresses (Fig. 2)

$$\sigma^*\Phi(\zeta)e^{i\gamma^0}+\alpha\overline{\Psi(\zeta)}e^{-i\gamma^0}=e^{i\gamma^0}[\underset{nn}{\sigma_{00}}(s^0)+i\underset{nt}{\sigma_{00}}(s^0)] \tag{2.2}$$

specified on the contour s^0 of the region, or for a specified configuration of the deformed contour s^0

$$\Phi(\zeta)e^{i\gamma^0}-\overline{\Psi(\zeta)}e^{-i\gamma^0}=-idz/ds^0 \tag{2.3}$$

Here and henceforth we will use complex coordinates and components of the vectors and tensors, introduced by the relations [13]

$$\begin{gathered}\zeta=x_1^0+ix_2^0,\quad z=x_1+ix_2\\ \frac{\partial}{\partial\zeta}=\frac{1}{2}\left(\frac{\partial}{\partial x_1^0}-i\frac{\partial}{\partial x_2^0}\right),\quad \frac{\partial}{\partial\bar{\zeta}}=\frac{1}{2}\left(\frac{\partial}{\partial x_1^0}+i\frac{\partial}{\partial x_2^0}\right)\\ T_1=t_{11}+t_{22}+i(t_{12}-t_{21}),\quad T_2=t_{11}-t_{22}+i(t_{12}+t_{21})\end{gathered} \tag{2.4}$$

The fundamental expressions defining the stress–strain state of the body in the plane problem of the non-linear theory of elasticity have the form [13]

$$\begin{gathered}z=\int\Phi(\zeta)d\zeta+\int\overline{\Psi(\zeta)d\zeta}\\ e^{2i\omega}=\Phi(\zeta)/\overline{\Phi(\zeta)},\quad \Delta=\Phi(\zeta)\overline{\Phi(\zeta)}-\Psi(\zeta)\overline{\Psi(\zeta)}\\ \Sigma_1^0=2\sigma^*[\Phi(\zeta)\overline{\Phi(\zeta)}]^{1/2},\quad \Sigma_2^0=2\alpha\overline{\Phi(\zeta)}\Psi(\zeta)[\Phi(\zeta)\overline{\Phi(\zeta)}]^{-1/2}\\ \lambda\Delta\Sigma_1=2[\sigma^*\Phi(\zeta)\overline{\Phi(\zeta)}]+\alpha\Psi(\zeta)\overline{\Psi(\zeta)}]\\ \lambda\Delta\Sigma_2=2[(\sigma^*+\alpha)\Phi(\zeta)\overline{\Psi(\zeta)}]\end{gathered} \tag{2.5}$$

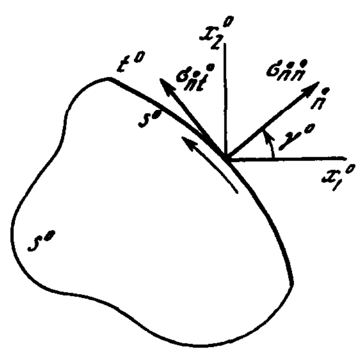

Fig. 2.

Here ω is the angle of rotation of a material particle, Δ is the multiplicity of the change in the area in the $x_3^0 = 0$ plane, λ is the multiplicity of the extension along the x_3^0 axis, and Σ_j^0 and Σ_j are the complex components of the tensors of the nominal and actual stresses, respectively.

It should be noted that, since the elastic constants σ_k^* and α_k of the contacting media are different, we need to obtain four functions of the complex variable $\Phi_k(\zeta)$, $\Psi_k(\zeta)$ $(k = 1, 2)$ in order to solve the problem completely. For each medium S_k there is a pair Φ_k, Ψ_k, which defines, by (2.5), the stress-strain state of this medium. The functions Φ_k, Ψ_k need only be holomorphic, generally speaking, in the corresponding regions S_k.

In addition to the above relations we also write the boundary conditions

$$\underset{1nn}{\sigma_{00}}(x_1^0) + i\underset{1nt}{\sigma_{00}}(x_1^0) = \underset{2nn}{\sigma_{00}}(x_1^0) + i\underset{2nt}{\sigma_{00}}(x_1^0), \quad \zeta \in L \tag{2.6}$$

$$dz_1 / dx_1^0 = dz_2 / dx_1^0, \quad \zeta \in L_2 \tag{2.7}$$

$$\underset{knn}{\sigma_{00}} = \underset{knt}{\sigma_{00}} = 0, \quad \zeta \in L_1, \quad k = 1, 2 \tag{2.8}$$

Here (2.6) is the condition of continuity of the stresses along L, (2.7) is the condition of continuity of the displacements in differential form along the part of the contact $L_2(x_1^0 = s^0)$, and (2.8) is the condition on the crack surface.

3. SOLUTION OF THE INITIAL PROBLEM

We will continue the function $\Phi_1(\zeta)$ analytically in S_2, and the function $\Phi_2(\zeta)$ analytically in S_1 as follows:

$$\Phi_k(\zeta) = (\alpha_k / \sigma_k^*)\overline{\Psi_k(\bar{\zeta})}, \quad \bar{\zeta} \in S_k, \quad k = 1, 2 \tag{3.1}$$

By (2.2) the functions $\Phi_k(\zeta)$ are continued continuously through the unloaded parts of the line L. Changing in (3.1) to conjugate quantities and substituting the result into (2.6) using (2.2) we obtain when $\gamma^0 = -\pi/2$

$$[\sigma_1^*\Phi_1(t) + \sigma_2^*\Phi_2(t)]^+ = [\sigma_1^*\Phi_1(t) + \sigma_2^*\Phi_2(t)]^-, \quad \zeta \in L \tag{3.2}$$

$$\Phi_k^{\pm}(t) = \lim_{\zeta \to t \pm i0} \Phi_k(\zeta)$$

It follows from (3.2) that the holomorphic function in the square brackets is constant over the whole plane, i.e.

$$\sigma_1^*\Phi_1(\zeta) + \sigma_2^*\Phi_2(\zeta) \equiv \sigma_1^* a_1^{\pm} + \sigma_2^* a_2^{\pm}, \quad a_k^{\pm} = \lim_{|t| \to \infty} \Phi_k^{\pm}(t)' \tag{3.3}$$

Then, the boundary conditions (2.7) and (2.8) reduce to the form

$$\Phi_1^+(t) + \beta\Phi_1^-(t) = Q, \quad \zeta \in L_2 \tag{3.4}$$

$$\Phi_1^+(t) - \Phi_1^-(t) = 0, \quad \zeta \in L_1 \tag{3.5}$$

$$\beta = \frac{1/\sigma_1^* + 1/\alpha_2}{1/\sigma_2^* + 1/\alpha_1}, \quad Q = \frac{1/\sigma_2^* + 1/\alpha_2}{\sigma_1^*(1/\sigma_2^* + 1/\alpha_1)}(\sigma_1^* a_1^{\pm} + \sigma_2^* a_2^{\pm})$$

respectively.

We will introduce the new function $F(\zeta)$, holomorphic on the line of contact and which vanishes at infinity

$$F(\zeta)=\begin{cases}\Phi_1(\zeta)-a_1^-, & \zeta\in S_1\\ -\beta^{-1}[\Phi_1(\zeta)-a_1^+], & \zeta\in S_2\end{cases} \tag{3.6}$$

From (3.4) and (3.5) we then obtain for the function $F(\zeta)$ a homogeneous Hilbert boundary-value problem, the general solution of which can be written in the form [14]

$$F(\zeta)=\frac{a_1^+-a_1^-}{2\pi i\beta X(\zeta)}\int_{-1}^{1}\frac{X(t)}{t-\zeta}dt+\frac{C}{X(\zeta)} \tag{3.7}$$

$$C=\text{const},\quad X(\zeta)=\sqrt{\zeta^2-1}\left(\frac{\zeta-1}{\zeta+1}\right)^{i\varepsilon},\quad \varepsilon=-\frac{\ln\beta}{2\pi}$$

$$X(t)=X^+(t)=-\beta X^-(t)=ie^{-\pi\varepsilon}\sqrt{1-t^2}\left(\frac{1-t}{1+t}\right)^{i\varepsilon},\quad |t|<1$$

Changing in (3.7) to integration over the closed contour, contractable to the section [–1, 1], and taking into account the expansion of the function $X(\zeta)$ at infinity, we obtain

$$F(\zeta)=\frac{(a_1^+-a_1^-)e^{\pi\varepsilon}}{2\operatorname{ch}(\pi\varepsilon)}\left\{1-\frac{\zeta-i2\varepsilon}{X(\zeta)}\right\}+\frac{C}{X(\zeta)} \tag{3.8}$$

We obtain the constant in (3.8) from the condition for the principal vector of all forces acting on an element of the lower half-plane $|x_1^0|<A$, $-B<x_2^0<0$ $(B>0)$ to be zero. As $A\to\infty$, $B\to\infty$ this condition leads to the equation

$$F_1+iF_2=\int_{-\infty}^{+\infty}(\sigma_{12}^{01\infty}+i\sigma_{22}^{01\infty})e^{i\omega_1^\infty}dt \tag{3.9}$$

where $(F_1,\ F_2)$ is the principal vector of the forces applied to L from the side of the positive direction of the x_2^0 axis.

Taking (3.1) and (3.6) into account, we have [1, 13]

$$F_1+iF_2=i\int_{-\infty}^{\infty}[\sigma_1^*\Phi_1^-(t)-\alpha_1\overline{\Psi}_1^-(t)]dt=i\sigma_1^*\int_{-\infty}^{+\infty}[2e^{\pi\varepsilon}\operatorname{ch}(\pi\varepsilon)F^-(t)+a_1^--a_1^+]dt \tag{3.10}$$

Substituting (3.10) and (3.7) into (3.9) we obtain $C=0$. In addition, we obtain from (2.5)

$$\frac{Q_1^k+Q_2^k}{2\sigma_k^*}e^{i\omega_k^\infty}=\begin{cases}a_1^-, & k=1\\ a_2^+, & k=2\end{cases} \tag{3.11}$$

$$\frac{Q_1^k-Q_2^k}{2\sigma_k^*}e^{i(\omega_k^\infty+2\chi_k)}=\begin{cases}a_1^+, & k=1\\ a_2^-, & k=2\end{cases}$$

We finally obtain

$$\Phi_k(\zeta)=\frac{1}{2\operatorname{ch}(\pi\varepsilon)}\left\{B_k+A_ke^{y}\frac{\zeta-i2\varepsilon}{X(\zeta)}\right\}$$

$$\Psi_k(\zeta)=\frac{\sigma_k^*}{2\alpha_k\operatorname{ch}(\pi\varepsilon)}\left\{\overline{B}_k-\overline{A}_ke^{-y}\frac{\zeta+i2\varepsilon}{X(\overline{\zeta})}\right\} \tag{3.12}$$

$$y=\pi\varepsilon(3-2k),\quad \zeta\in S_k$$

$$A_k=\frac{\sigma_{22}^{0k\infty}-i\sigma_{12}^{0k\infty}}{\sigma_k^*}e^{i\omega_k^\infty}$$

$$B_k=\frac{\operatorname{ch}(\pi\varepsilon)\sigma_{11}^{0k\infty}+(-1)^k\operatorname{sh}(\pi\varepsilon)\sigma_{22}^{0k\infty}+ie^{y}\sigma_{12}^{0k\infty}}{\sigma_k^*}e^{i\omega_k^\infty}$$

($\sigma_{ij}^{0k\infty}$ are the values of the nominal stresses σ_{ij}^0 at infinity in the medium S_k).

Since an infinitely distant point is situated on the line of contact L_2, the displacements and stresses at infinity must satisfy the contact conditions (2.6) and (2.7) or (3.2) and (3.4). In the limit in (3.2) and (3.4) as $x_1^0 \to \infty$ taking (3.11) into account, we obtain

$$(\sigma_{12}^{01\infty} + i\sigma_{22}^{01\infty})e^{i\omega_1^\infty} = (\sigma_{12}^{02\infty} + i\sigma_{22}^{02\infty})e^{i\omega_2^\infty} \tag{3.13}$$

$$\begin{aligned}&(1/\sigma_2^* + 1/\alpha_2)(\sigma_{11}^{02\infty} - \sigma_{22}^{02\infty} + i\sigma_{12}^{02\infty})e^{i(\omega_2^\infty - \omega_1^\infty)} = \\ &= (1/\sigma_1^* + 1/\alpha_1)(\sigma_{11}^{01\infty} + \sigma_{22}^{01\infty}) - 2(1/\sigma_2^* + 1/\alpha_2)(\sigma_{22}^{01\infty} - i\sigma_{12}^{01\infty})\end{aligned} \tag{3.14}$$

respectively.

Relations (3.13) and (3.14) establish a connection between the stresses applied to the lower and upper half-planes at infinity. In particular, when

$$\sigma_{12}^{0k\infty} = \sigma_{22}^{0k\infty} = 0 \quad (k = 1,2) \tag{3.15}$$

it follows from (3.14) that

$$\omega_1^\infty = \omega_2^\infty, \quad (1/\sigma_2^* + 1/\alpha_2)\sigma_{11}^{02\infty} = (1/\sigma_2^* + 1/\alpha_1)\sigma_{11}^{01\infty} \tag{3.16}$$

The stressed state, defined by (3.15) and (3.16), is characteristic of the fact that the occurrence of a crack between two media has no effect on it. In this degenerate case, the complex potentials Φ_k and Ψ_k have constant values

$$\Phi_k = \frac{1/\sigma_1^* + 1/\alpha_1}{1/\sigma_k^* + 1/\alpha_k}H, \quad \Psi_k = \frac{\sigma_k^*}{\alpha_k^*}\overline{\Phi}_k \quad (\zeta \in S_k) \tag{3.17}$$

$$H = [\sigma_{11}^{01\infty}/(2\sigma_1^*)]e^{i\omega_1^\infty}$$

On the other hand, when $\omega_1^\infty = \omega_2^\infty$, we have

$$\sigma_{22}^{0k\infty} = \sigma_{22}^{0\infty}, \quad \sigma_{12}^{0k\infty} = \begin{cases}\sigma_{12}^{0\infty}, & \alpha_1 = \alpha_2 \\ 0, & \alpha_1 \neq \alpha_2\end{cases} \tag{3.18}$$

$$(1/\sigma_2^* + 1/\alpha_2)\sigma_{11}^{02\infty} = (1/\sigma_1^* + 1/\alpha_1)\sigma_{11}^{01\infty} + (1/\sigma_1^* - 1/\sigma_2^* + 1/\alpha_2 - 1/\alpha_1)\sigma_{22}^{0\infty}$$

Moreover, when a preliminary uniform tension $\sigma_{11}^{01\infty} = \sigma_{22}^{01\infty} = \sigma_1^*$, $\sigma_{12}^{02\infty} = 0$ acts in S_1 far from the crack and also $\sigma_{12}^{02\infty} = 0$, from (3.13) we obtain $\omega_2^\infty = \omega_1^\infty$, $\sigma_{22}^{02\infty} = \sigma_1^*$. It then follows from (3.14) that

$$\sigma_{11}^{02\infty} = \sigma_1^* + \frac{2(\sigma_2^* - \sigma_1^*)}{\sigma_2^*(1/\sigma_2^* + 1/\alpha_2)} \tag{3.19}$$

We see from (3.19) that only when $\sigma_2^* = \sigma_1^*$ in both media, far from the crack, can one obtain a preliminary uniform tension σ_1^*, i.e. the initial undeformed state.

When there is no crack, i.e. when there is continuous contact between the two different media, it obviously follows from (2.1), (3.7) and (3.8) that

$$\begin{aligned}&\Phi_1 = a_1^-, \quad \Psi_1 = (\sigma_1^*/\alpha_1)\bar{a}_1^+ \quad (\zeta \in S_1) \\ &\Phi_2 = a_2^+, \quad \Psi_2 = (\sigma_2^*/\alpha_2)\bar{a}_2^- \quad (\zeta \in S_2)\end{aligned} \tag{3.20}$$

The stressed state in each medium is then constant and is identical with the stressed state at infinity. However, if there are no deformations when a uniform tension $\sigma_{11}^{01}=\sigma_{22}^{01}=\sigma_1^*$ acts in S_1, the medium S_2 will in this case undergo a non-zero deformation when $\sigma_2^*\neq\sigma_1^*$.

We obtain the position of the points of each medium, from the first relation in (2.5) and (3.12)

$$z_k=\frac{A_k}{2\operatorname{ch}(\pi\varepsilon)}\left[e^{\gamma}\frac{\zeta^2-1}{X(\zeta)}-e^{-\gamma}\frac{\sigma_k^*}{\alpha_k}\frac{\bar\zeta^2-1}{X(\bar\zeta)}\right]+\frac{B_k}{2\operatorname{ch}(\pi\varepsilon)}\left[\zeta+\frac{\sigma_k^*}{\alpha_k}\bar\zeta\right],\quad \zeta\in S_k \tag{3.21}$$

We obtain the deformation of the crack surface from (3.21) with $\zeta=x_1^0$, $|x_1^0|\leqslant 1$

$$z_k(x_1^0)=\frac{\sigma_k^*(1/\sigma_k^*+1/\alpha_k)}{2\operatorname{ch}(\pi\varepsilon)}\left\{B_kx_1^0+i(-1)^kA_k\sqrt{1-(x_1^0)^2}\left(\frac{1+x_1^0}{1-x_1^0}\right)^{i\varepsilon}\right\} \tag{3.22}$$

We see from (3.22) that even in the simplest case of tension at infinity $\sigma_{jj}^{0k\infty}=\sigma^*$, $\sigma_{12}^{0k\infty}=\omega_k^\infty=0$ $(j,\ k=1,\ 2)$ when $\sigma_1^*=\sigma_2^*=\sigma^*$ the surfaces of the cracks penetrate into one another in the region of the tip. In fact, in this cases $A_k=1$, $B_k=e^{\pi\varepsilon(2k-3)}$ and

$$z_k(x_1^0)=$$
$$=\frac{\sigma^*(1/\sigma^*+1/\alpha_k)}{2\operatorname{ch}(\pi\varepsilon)}\left\{x_1^0e^{-\gamma}-(-1)^k\sqrt{1-(x_1^0)^2}\left[\sin\left(\varepsilon\ln\frac{1+x_1^0}{1-x_1^0}\right)-i\cos\left(\varepsilon\ln\frac{1+x_1^0}{1-x_1^0}\right)\right]\right\} \tag{3.23}$$

As one approaches the right-hand tip $(x_1^0\to 1)$ the first points of the crack surfaces, corresponding to maximum "overlap", have the coordinate $x_1^0=th(\tfrac{1}{2}\pi/\varepsilon)$. In this case

$$x_1^k=\frac{\sigma^*(1/\sigma^*+1/\alpha_k)e^{-\gamma}}{2\operatorname{ch}(\pi\varepsilon)\operatorname{cth}(\tfrac{1}{2}\pi/\varepsilon)}$$
$$x_2^k=(-1)^{k+1}=\frac{\sigma^*(1/\sigma^*+1/\alpha_k)}{2\operatorname{ch}(\pi\varepsilon)\operatorname{ch}(\tfrac{1}{2}\pi/\varepsilon)} \tag{3.24}$$

Although the displacements of these points along L_1 are not identical $(x_1^1\neq x_1^2)$, it can be shown that, in general, when $\alpha_1\neq\alpha_2$ this does not prevent mutual penetration of the crack surfaces. On the other hand, when $1/\sigma_1^*+1/\alpha_2=1/\sigma_2^*+1/\alpha_1$ there are no oscillations of the displacements.

Using (2.5) and (3.12) we can also determine the stressed state at any point of each medium. Thus, in the neighbourhood of the right-hand tip of the crack $(x_1^0=1)$ we have the asymptotic forms for the compressed materials of both media

$$\frac{\partial z_k}{\partial\zeta}=\Phi_k(\zeta)\sim\frac{r^{0-i\varepsilon}}{2\sigma_k^*\sqrt{2\pi r^0}}Kf_ke^{-i\varphi^0/2}$$

$$\frac{\partial z_k}{\partial\bar\zeta}=\overline{\Psi_k(\zeta)}\sim-\frac{r^{0-i\varepsilon}}{2\alpha_k\sqrt{2\pi r^0}}Kf_k^{-1}e^{i\varphi^0/2}$$

$$f_k=\exp[\pi\varepsilon(3-2k)+\varepsilon\varphi^0],\quad k=1,2$$

$$K=K_1-iK_2=2\sqrt{2\pi}e^{-\pi\varepsilon}\lim_{\zeta\to 1}(\zeta-1)^{1/2+i\varepsilon}\Phi_k(\zeta)=\frac{(1-i2\varepsilon)\sqrt{\pi}e^{i\varepsilon\ln 2}}{\operatorname{ch}(\pi\varepsilon)}(\sigma_{22}^{0k\infty}-i\sigma_{12}^{0k\infty})e^{i\omega_k^\infty}$$

Here

$$\Sigma_1^{0k} \sim \frac{|K|}{\sqrt{2\pi r^0}} f_k, \quad \Sigma_2^{0k} \sim -\frac{|K|}{\sqrt{2\pi r^0}} f_k^{-1} e^{i\varphi^0}$$
$$\Delta_k \sim \frac{|K|^2}{4\sigma_k^{*2}(2\pi r^0)}\left[f_k^2 - \left(\frac{\sigma_k^*}{\alpha_k f_k}\right)^2\right] \tag{3.25}$$

Then

$$\frac{\lambda \Sigma_1^k}{\sigma_k^*} \sim 2\frac{f_k^4 + \sigma_k^* / \alpha_k}{\psi}$$
$$\frac{\lambda \Sigma_2^k}{\sigma_k^*} \sim \frac{2K(1+\sigma_k^*/\alpha_k) f_k^2}{\bar{K}\psi} r^{0^{-i2\varepsilon}} \tag{3.26}$$
$$\psi = f_k^4 - (\sigma_k^* / \alpha_k)^2$$

The asymptotic relations obtained show that under conditions of plane deformation ($\lambda = \text{const}$) of compressed materials the actual stresses in the neighbourhood of the tip of the crack are limited and have an oscillatory form defined by the factor $r^{0^{-i2\varepsilon}}$. At the same time, the conventional stresses have the same singularity as the stresses in the linearly elastic problem, but, unlike the latter, they do not oscillate as $r^0 \to 0$. On changing to a polar system of coordinates r^0, φ^0 (Fig. 1) from (3.26) and the equalities

$$\sigma^0_{r^0 r^0} + \sigma^0_{\varphi^0\varphi^0} = \Sigma_1^0, \quad \sigma^0_{r^0 r^0} - \sigma^0_{\varphi^0\varphi^0} + i2\sigma^0_{r^0\varphi^0} = \Sigma_2^0 e^{-i2\varphi^0}$$

we obtain for the nominal stresses

$$\sigma^{0k}_{r^0 r^0} \sim \frac{|K|}{2\sqrt{2\pi r^0}}(f_k - f_k^{-1}\cos\varphi^0)$$
$$\sigma^{0k}_{\varphi^0\varphi^0} \sim \frac{|K|}{2\sqrt{2\pi r^0}}(f_k + f_k^{-1}\cos\varphi^0) \tag{3.27}$$
$$\sigma^{0k}_{r^0\varphi^0} \sim \frac{|K|}{2\sqrt{2\pi r^0}} f_k^{-1}\sin\varphi^0$$

From the asymptotic representations of the quantities $\partial z_k / \partial\zeta$ and $\partial z_k / \partial\bar{\zeta}$ we also obtain [13] the value of the Rice–Cherepanov integral

$$J = \tfrac{1}{8}|K|^2(1/\sigma_1^* + 1/\alpha_1 + 1/\sigma_2^* + 1/\alpha_2)$$

4. SOLUTION OF THE LINEAR PROBLEM

We will consider the solution of the plane problem of the linear theory of elasticity for a crack at the interface between two media which, for small strains and angles of rotation, correspond to Hooke's law (1.1). We will assume that the value of the preliminary uniform tension in each medium is equal to σ_k^* and is an independent parameter. In view of the smallness of the strains and angles of rotation, we will omit the superscript 0 everywhere.

By [13, 14], the expressions for the stresses and displacements in the medium S_k, using the notation (2.4), take the form

$$\Sigma_1^k + i\frac{4\alpha_k^{\upsilon}}{\kappa_k^{\upsilon}+1}\omega_k = 4\Phi_k(\zeta)+2\sigma_k^r$$

$$(\Sigma_1^k - \Sigma_2^k)/2 = \Phi_k(\zeta) - \Phi_k(\bar{\zeta}) + (\zeta - \bar{\zeta})\overline{\Phi_k'(\zeta)} + \sigma_k^r \qquad (4.1)$$

$$\alpha_k^{\upsilon}(u_1^k + iu_2^k) = \kappa_k^{\upsilon}\int\Phi_k(\zeta)d\zeta + \int\Phi_k(\bar{\zeta})d\bar{\zeta} - (\zeta-\bar{\zeta})\overline{\Phi_k(\zeta)} \qquad (4.2)$$

where u_j^k is the component of the displacement vector of points of the medium S_k along the x_j axis, $\kappa_k^{\upsilon} = 1 + 2\alpha_k^{\upsilon}/\sigma_k^{\upsilon}$ for plane strain and $\kappa_k^{\upsilon} = (5/\alpha_k^{\upsilon} + 1/\sigma_k^{\upsilon})/(3/\alpha_k^{\upsilon} - 1/\sigma_k^{\upsilon})$ for the plane stressed state, $j,\ k = 1, 2$.

Conditions (2.6)–(2.8) in this case can be written in the form

$$\Sigma_1^1 - \Sigma_2^1 = \Sigma_1^2 - \Sigma_2^2, \quad \zeta \in L \qquad (4.3)$$

$$u_{1,1}^1 + iu_{2,1}^1 = u_{1,1}^2 + iu_{2,1}^2, \quad \zeta \in L_2 \qquad (4.4)$$

$$\Sigma_1^k - \Sigma_2^k = 0, \quad \zeta \in L_1, \quad (k = 1,2) \qquad (4.5)$$

respectively.

It follows from (4.1) and (4.3) that

$$[\Phi_1(t) + \Phi_2(t)]^+ - [\Phi_1(t) + \Phi_2(t)]^- = \sigma_1^r - \sigma_2^r, \quad \zeta \in L \qquad (4.6)$$

This means that

$$\Phi_1(\zeta) + \Phi_2(\zeta) = \begin{cases} a_1^+ + a_2^+, & \zeta \in S_1 \\ a_1^- + a_2^- & \zeta \in S_2 \end{cases} \qquad (4.7)$$

where

$$a_1^- - a_1^+ + \sigma_1^r = a_2^+ - a_2^- + \sigma_2^r \qquad (4.8)$$

By (4.1) and (4.8) we have

$$\sigma_{22}^{1\infty} - i\sigma_{12}^{1\infty} = \sigma_{22}^{2\infty} - i\sigma_{22}^{2\infty} \equiv \sigma_{22}^{\infty} - i\sigma_{12}^{\infty} \qquad (4.9)$$

$$\frac{\sigma_{11}^{k\infty} + \sigma_{22}^{\infty} - 2\sigma_k^r}{4} + i\frac{\omega_k^{\infty}}{\kappa_k^{\upsilon}/\alpha_k^{\upsilon} + 1/\alpha_k^{\upsilon}} = \begin{cases} a_1^-, & k=1 \\ a_2^+, & k=2 \end{cases} \qquad (4.10)$$

$$a_1^- - a_2^+ + \sigma_1^r = \sigma_{22}^{\infty} - i\sigma_{12}^{\infty}$$

It is appropriate to note here that the solution of the problem in the linear formulation requires continuity of the stresses σ_{22} and σ_{12} on the section where the two media are in contact, including at infinity, whereas in the non-linear problem the nominal stresses σ_{22}^0 and σ_{12}^0 are discontinuous when $\omega_1^{\infty} \neq \omega_2^{\infty}$ (it is sufficient to compare (3.13) and (4.9)).

When Eqs (4.7) are taken into account, we convert conditions (4.4) and (4.5) respectively to the boundary conditions for the single function $\Phi_1(\zeta)$

$$\Phi_1^+(t) + \eta\Phi_1^-(t) = M, \quad \zeta \in L_2 \qquad (4.11)$$

$$\Phi_1^+(t) - \Phi_1^-(t) = \sigma_1^r, \quad \zeta \in L_1 \qquad (4.12)$$

$$\eta = (\kappa_1^{\upsilon}/\alpha_1^{\upsilon} + 1/\alpha_2^{\upsilon})/(\kappa_2^{\upsilon}/\alpha_2^{\upsilon} + 1/\alpha_1^{\upsilon})$$

$$M=\frac{1/\alpha_2^{\upsilon}}{\kappa_2^{\upsilon}/\alpha_2^{\upsilon}+1/\alpha_1^{\upsilon}}[\kappa_2^{\upsilon}(a_1^+ +a_2^+)+a_1^- +a_2^-]$$

For the function $F(\zeta)$, defined by (3.6) (with β replaced by η), from (4.11) and (4.12) we also obtain a non-homogeneous Hilbert boundary-value problem, the solution of which has the form

$$F(\zeta)=\frac{(a_1^+ -a_1^- -\sigma_1^r)e^{\pi\varepsilon_e}}{2\,\mathrm{ch}(\pi\varepsilon_e)}\left[1-\frac{\zeta-i2\varepsilon_e}{Y(\zeta)}\right] \tag{4.13}$$

$$\varepsilon_e=-\frac{\ln\eta}{2\pi},\qquad Y(\zeta)=\sqrt{\zeta^2-1}\left(\frac{\zeta-1}{\zeta+1}\right)^{i\varepsilon_e}$$

Further, from (3.6) and (4.7) we have

$$\Phi_k(\zeta)=\frac{1}{2\,\mathrm{ch}(\pi\varepsilon_e)}\left[B_k+\delta De^{\delta y_e}\frac{\zeta-i2\varepsilon_e}{Y(\zeta)}\right]+\frac{1}{2}(1-\delta)\sigma_k^r \tag{4.14}$$

$$\delta=\begin{cases}1, & \zeta\in S_k\\ -1, & \zeta\in \bar{S}_k\end{cases},\qquad y_e=\pi\varepsilon_e(3-2k),\qquad k=1,2$$

$$B_k=\frac{\mathrm{ch}(\pi\varepsilon_e)\sigma_{11}^{k\infty}+\{\mathrm{sh}\,y_e-e^{y_e}\}\sigma_{22}^{\infty}}{2}-\mathrm{ch}(\pi\varepsilon_e)\sigma_k^r+i\left[e^{y_e}\sigma_{12}^{\infty}+\frac{2ch(\pi\varepsilon_e)}{\kappa_k^{\upsilon}/\alpha_k^{\upsilon}+1/\alpha_k^{\upsilon}}\omega_k^{\infty}\right]$$

$$D=\sigma_{22}^{\infty}-i\sigma_{12}^{\infty}$$

Letting $x_1\to\infty$ in (4.11) and separating the real and imaginary parts, using (4.9) and (4.10) we obtain relations similar to (3.14) in the non-linear problem

$$\sigma_{11}^{2\infty}-2\frac{\kappa_2^{\upsilon}-1}{\kappa_2^{\upsilon}+1}\sigma_2^r=\mu\left(\sigma_{11}^{1\infty}-2\frac{\kappa_1^{\upsilon}-1}{\kappa_1^{\upsilon}+1}\sigma_1^r\right)+\frac{3+\mu-(3\mu+1)e^{2\pi\varepsilon_e}}{2ch(\pi\varepsilon_e)e^{\pi\varepsilon_e}}\sigma_{22}^{\infty} \tag{4.15}$$

$$\omega_2^{\infty}-\omega_1^{\infty}=\frac{\alpha_2^{\upsilon}-\alpha_1^{\upsilon}}{\alpha_1^{\upsilon}\alpha_2^{\upsilon}}\sigma_{12}^{\infty} \tag{4.16}$$

$$\mu=(\kappa_1^{\upsilon}/\alpha_1^{\upsilon}+1/\alpha_1^{\upsilon})/(\kappa_2^{\upsilon}/\alpha_2^{\upsilon}+1/\alpha_2^{\upsilon})$$

From (4.14)–(4.16) we obtain the stresses for which the crack does not open

$$\sigma_{12}^{\infty}=\sigma_{22}^{\infty}=0 \tag{4.17}$$

$$\sigma_{11}^{2\infty}-2\frac{\kappa_2^{\upsilon}-1}{\kappa_2^{\upsilon}+1}\sigma_2^r=\mu\left(\sigma_{11}^{1\infty}-2\frac{\kappa_1^{\upsilon}-1}{\kappa_1^{\upsilon}+1}\sigma_1^r\right)$$

In this degenerate case Φ_k have the following constant values

$$\Phi_1=\frac{\sigma_{11}^{1\infty}-2\sigma_1^r}{4}+i\frac{\omega_1^{\infty}}{\kappa_1^{\upsilon}/\alpha_1^{\upsilon}+1/\alpha_1^{\upsilon}}+\frac{1}{2}(1-\delta)\sigma_1^r$$

$$\Phi_2=\mu\Phi_1+\frac{\sigma_1^r/\alpha_1^{\upsilon}-\sigma_2^r/\alpha_2^{\upsilon}}{\kappa_2^{\upsilon}/\alpha_2^{\upsilon}+1/\alpha_2^{\upsilon}}+\frac{1}{2}(1-\delta)\sigma_2^r-\frac{\mu}{2}(1+\delta)\sigma_1^r \tag{4.18}$$

Note that when $\sigma_k^r = 0$ $(k = 1, 2)$ relations (4.14)–(4.18) are identical with the corresponding expressions obtained in [6].

Suppose that a preliminary uniform tension $\sigma_{11}^{1\infty} = \sigma_{22}^{\infty} = \sigma_1^r$, $\sigma_{12}^{\infty} = 0$ acts in S_1 at infinity. We then obtain from (4.15)

$$\sigma_{11}^{2\infty} = \sigma_1^r + 2\frac{\kappa_2^\upsilon - 1}{\kappa_2^\upsilon + 1}(\sigma_2^r - \sigma_1^r) \tag{4.19}$$

whence it follows that, as in the non-linear problem, a preliminary uniform tension corresponding to zero strain can occur far from the crack simultaneously both in S_1 and S_2 only when $\sigma_2^r = \sigma_1^r$.

In the case when there is continuous contact between the media the stressed state in each medium is identical with the corresponding stressed state at infinity, while the function Φ_k is such that

$$\begin{aligned} &\Phi_1 = a_1^-, \quad \Phi_2 = a_2^+ - \sigma_{22}^\infty + i\sigma_{12}^\infty + \sigma_2^r \quad (\zeta \in S_1) \\ &\Phi_1 = a_1^- - \sigma_{22}^\infty + i\sigma_{12}^\infty + \sigma_1^r, \quad \Phi_2 = a_2^+ \quad (\zeta \in S_2) \end{aligned} \tag{4.20}$$

The displacements of the points of each medium can be found from (4.2) using (4.14)

$$\begin{aligned} \alpha_k^\upsilon(u_1^k + iu_2^k) = {} & \frac{1}{2\operatorname{ch}(\pi\varepsilon_e)}\left\{ D\left[\kappa_k^\upsilon e^{\gamma_e}\frac{\zeta^2 - 1}{Y(\zeta)} - e^{-\gamma_e}\frac{\bar\zeta^2 - 1}{\overline{Y(\zeta)}}\right] + B_k(\kappa_k^\upsilon\zeta + \bar\zeta) - \right. \\ & \left. -(\zeta - \bar\zeta)\left[\bar B_k + \bar D e^{\gamma_e}\frac{\bar\zeta + i2\varepsilon_e}{\overline{Y(\zeta)}}\right]\right\} + \sigma_k^r\zeta, \quad \zeta \in S_k \end{aligned} \tag{4.21}$$

Hence we obtain expressions for the displacements of points of crack surface when $\zeta = x_1$, $|x_1| < 1$

$$\alpha_k^\upsilon[(u_1^k(x_1) + iu_2^k(x_1)] = \frac{\kappa_k^\upsilon + 1}{2\operatorname{ch}(\pi\varepsilon_e)}\left[B_k x_1 + i(-1)^k D\sqrt{1 - x_1^2}\left(\frac{1 + x_1}{1 - x_1}\right)^{i\varepsilon_e}\right] + \sigma_k^r x_1, \quad k = 1, 2 \tag{4.22}$$

In the neighbourhood of the right tip of the crack when $\zeta \to 1$ we have from (4.1) and (4.14) the following asymptotic forms of the linear problem

$$\begin{aligned} &\Phi_k(\zeta) \sim \frac{r^{-i\varepsilon_e}}{2\sqrt{2\pi r}}\delta K_e g_k e^{-i\varphi/2} \\ &\Phi_k'(\zeta) \sim -\frac{r^{-i\varepsilon_e}}{2\sqrt{2\pi r}}\delta K_e(1 + i2\varepsilon_e)g_k e^{-i3\varphi/2} \\ &g_k = \exp[\delta\pi\varepsilon_e(3 - 2k) + \varepsilon_e\varphi], \quad k = 1, 2 \\ &K_e = K_{1e} - iK_{2e} = 2\sqrt{2\pi}e^{-\pi\varepsilon_e}\lim_{\zeta\to 1}(\zeta - 1)^{1/2 + i\varepsilon_e}\Phi_k(\zeta) = \frac{(1 - i2\varepsilon_e)\sqrt{\pi}e^{i\varepsilon_e \ln 2}}{\operatorname{ch}(\pi\varepsilon_e)}(\sigma_{22}^\infty - i\sigma_{12}^\infty) \end{aligned} \tag{4.23}$$

Then, we obtain the following expressions for the complex components of the stress tensor

$$\begin{aligned} &\Sigma_1^k \sim \frac{2}{\sqrt{2\pi r}}g_k \operatorname{Re}\{K_e r^{-i\varepsilon_e}e^{-i\varphi/2}\} \\ &\Sigma_2^k \sim \frac{2}{\sqrt{2\pi r}}\{K_e g^{-1}{}_k r^{-i\varepsilon_e}e^{-i\varphi/2} - \bar K_e[1 + i(1 - i2\varepsilon_e)\sin\varphi e^{i\varphi}]g_k r^{i\varepsilon_e}e^{i\varphi/2}\}, \quad \zeta \in S_k \end{aligned} \tag{4.24}$$

From (4.24) it is fairly easy to obtain relations for the components of the stresses in a polar system of coordinates. When $\sigma_k^r = 0$ $(k = 1, 2)$ these relations are identical with the well-known asymptotic forms of the linear problem [15], since in this case the constants λ_k^υ and μ_k^υ, in terms of which σ_k^υ and σ_k^υ, are expressed, are identical with the corresponding Lamé parameters $\lambda_{(k)}$ and $\mu_{(k)}$. The same can be said of the Rice–Cherepanov integral of the linear problem, for which, for an arbitrary value of σ_k^υ, the following expression holds

$$J_e = \tfrac{1}{4}|K_e|^2 (1/\sigma_1^\upsilon + 1/\sigma_2^\upsilon + 1/\alpha_1^\upsilon + 1/\alpha_2^\upsilon)$$

It should be noted, however, that the dependence of the asymptotic forms (4.24) on the parameters σ_k^r is related to the particular feature of the interface crack itself. For a crack in a uniform medium $(\varepsilon_e = 0)$ relations (4.24) are independent of the value of the residual stress σ^r $(\sigma^r = \sigma_1^r = \sigma_2^r)$. The asymptotic forms of the non-linear problem obviously possess the same property. Although expressions (3.27) were obtained for specific values of σ_k^r, their dependence on the elastic constants does not differ from the similar dependence of the asymptotic forms of the linear problem (4.24).

Note also that for any values of σ_k^r the strains and angles of rotation are small in the region where the stress–strain state differs only slightly from the initial state, i.e. it corresponds to a configuration close to the reference configuration. This state is reached, for example, far from the crack when $\sigma_k^r = \sigma^r$, $\sigma_{jj}^{0k\infty} = \sigma^r$, $\sigma_{12}^{0k\infty} = \omega_k^\infty = 0$ $(k,\ j = 1,\ 2)$. Here, since the surfaces of the crack are free from external loads, the stress–strain state in the neighbourhood of the middle of the crack will differ only slightly from the initial state if $\sigma^r/\sigma_k^\upsilon \ll 1$ $(k = 1,\ 2)$. Otherwise (particularly when $\sigma_k^r = \sigma_k^\upsilon = \sigma_k^*$) the crack as a whole will be situated in a region of large strains and an analysis of the behaviour of the crack is only possible by solving the corresponding non-linear problem.

5. CONCLUSIONS

The solution of the problem in its non-linear form revealed certain advantages of the nominal stresses over the stresses of the linear problem, and also over the actual stresses. This is discussed in detail in [13, 16].

The nominal stresses do not oscillate as one approaches the tip of the crack, which enables one to justify physically their use in the force criterion of fracture. At the same time, the use of the actual stresses to predict fracture is problematical in view of their limited nature and oscillatory form close to the tip of the crack.

An important feature of an interface crack is the fact that the stress-intensity factors K_j (K_{je}) for them do not have any physical meaning, since they depend both on the normal and on the tangential forces. The nominal stresses have a clear advantage in this respect over the stresses of the linear problem, since the asymptotic formulae for the first (unlike the asymptotic forms of the linear problem) contain the same general load parameter $|K|$ as the integral J. Hence, for the conventional stresses it obviously follows that the force and energy criteria of fracture are equivalent not only for any one type of fracture, but also for the mixed type of fracture (normal cleavage and shear simultaneously).

The conventional stresses and the stresses of the linear problem have different asymptotic dependences on the polar angle. This raises doubts about the results obtained by analysing the asymptotic relationships of the linear problems if the dimensions of the regions of high elastic deformation around the tip of the crack exceed the limits allowed by linear fracture mechanics.

A characteristic feature of the solutions of the non-linear and linear problems is the oscillations of the displacements as one approaches the tip of the crack, and, as a consequence of this, the interpenetration of the surfaces of the crack. In general, the parameters of the oscillation ε and ε_e differ, but for plane strain and when the shear moduli of both media are the same $(\alpha_1 = \alpha_2)$, they are equal. In the latter case some results obtained for the linear problem can easily be transferred to the non-linear problem. In particular, for pure stretching, consideration of the contact between the crack surfaces in the linear problem does not lead to any appreciable corrections, while the dimensions of the contact zone are practically identical with the small dimensions where, according to the solutions of the problems, the surfaces of the crack penetrate into one another [11]. At the same time, when shear stresses σ_{12}^{∞} act on the material the contact zone in the linear problem may be large, and the solution with oscillations is inadmissible [11, 12]. Nevertheless, as follows from [17, 18], the ideas of linear fracture mechanics can nevertheless also be used in the case of a stress σ_1^{∞}. Here the zone of penetration of the surfaces of the crack must have dimensions of $r_c < 0.01$ [17], which, for the most widely used composites with oscillation parameters $|\varepsilon_\epsilon| < 0.15$ [18], leads to the inequality $|\sigma_{12}^{\infty}| < 0,\ 84\sigma_{22}^{\infty}$. A similar result would be expected for the non-linear problem also. At least it is obvious when $\varepsilon = \varepsilon_\epsilon$.

When the region of oscillation has considerable dimensions it is necessary to consider the corresponding non-linear problem taking contact between the crack surfaces into account. When there are no limitations on the value of the elastic strain its solutions will be free from the contradictions which characterize the similar solution to the linear problem.

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